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Study on Radiation in 3-D Irregular Systems Using the Trapezoidal Rule Approximation on the Transport Equation

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In this study, the integral of the radiative source function appearing in the apparent solution of the Radiative Transfer Equation (RTE) is approximated by the two-point trapezoidal rule which is different from the Taylor series expansion approximation used in the Finite Volume Method (FVM) or the discrete ordinates interpolation method. The resulting equation derived from the trapezoidal rule approximation has much simpler form than that obtained from the existing Taylor series approximation. The approximate equation of transfer by the trapezoidal rule is applied for the discrete ordinates interpolation method using nonorthogonal grid systems to predict the radiative heat transfer in 3-D enclosures filled with a gray, absorbing, emitting and scattering medium. The upstream intensity and the source function required for analyses are determined by a linear interpolation on a diagonally placed triangular plane that is reported to be very simple for radiative transfer analyses in three dimensional irregular systems. Numerical results indicate that good accuracy is obtained by using the trapezoidal rule which showed fairly good agreement with the results from the zone method or the original discrete ordinates interpolation method both of which are considered to be more accurate as compared to the conventional S-N discrete ordinate method and the FVM. The trapezoidal rule proposed in this study is successful for nonorthogonal grid systems and it can be used for analyses of the radiative transfer in three dimensional irregular enclosures.

Key Words: Radiation, DOIM(Discrete Ordinates Interpolation Method)Irregular Geometry, Trapezoidal Rule, Mie-Anisotropic Scattering

1. Introduction

Recent energy systems are complicated in their shapes to achieve compactness and high thermal efficiency. As a result, system boundaries are often irregular and the nonorthogonal computational grids are frequently used to adapt to the irregular boundary area faces. The solution methods for convective hear transfer using nonorthogonal grids are fairly well developed and many commercial codes are appearing in the public domain (Peric, 1985). However, for radiative heat transfer analyses, solution methods are under development for the nonorthogonal grids to be applied in irregular systems.

Chui and Raithby (1993) proposed a solution method for nonorthogonal grids by using the finite-volume technique to solve the radiative transfer in 2D four-sided enclosure, annular cylinder and J-shaped enclosure filled with absorbing, emitting and scattering media. Chai et al. (1994) proposed a somewhat different finite volume method where they used a formulation similar to the ordinary discrete ordinates method, and the method was successfully applied for some 2D irregular enclosures. Kim and Baek (1997) applied the finite volume method of Chui and Raithby for radiative transfer analyses in three dimensional enclosures such as a regular parallelepiped and an irregular gas turbine shaped system. On the other hand, Cheong and Song (1996) proposed a new scheme for the radiative

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transfer in 2D irregular enclosures where they tried to compute the radiative intensities at the point in the medium directly without considering the usual control volume concept and they called this method as the discrete ordinates interpolation method. Seo and Kim (1996) extended the discrete ordinates interpolation method to the three dimensional irregular systems by using a simple yet accurate interpolation scheme where they proposed a linear interpolation on the diagonally placed triangular interpolation plane. The proposed scheme has proved to be successful for highly irregular three dimensional systems. It is notable that all of the solution methods for irregular systems outlined above are developed by using the integrated apparent solution of the radiative transfer equation.

In this study, the integrated apparent solution of the radiative transfer equation is re-examined for a suitable approximation to be applied for the FVM or the discrete ordinates interpolation method developed for irregular systems. Previous approximation to the apparent solution has relied on the Taylor series expansion on the radiative source function and then a spatial integration performed analytically, resulting in a more or less complicated final equation for the radiative intensity. The motivation of this paper is to find a simpler form of the final approximated equation from the original apparent solution. For this purpose, the trapezoidal rule is directly applied to the spatial integration and the performance of this approximation is studied by applying it to some three dimensional regular and irregular enclosures using the discrete ordinates interpolation method.

2. Theoretical Background

2.1 Radiative transfer equation

For an absorbing, emitting and scattering medium the radiative transfer equation can be written by donsidering a ray traveling along a path s in the direction of Ω through the medium as (Siegel and Howell, 1992; Kim, 1995)

$$\frac{\partial I(s,\ \Omega)}{\partial s}+\beta(s)I(s,\ \Omega)=S(s,\ \Omega) \quad (1a)$$

where the radiative source function $S(s,\ \Omega)$ is defined as

$$S(s,\ \Omega)=a(s)I_b(s)+\frac{\sigma_s(s)}{4\pi}\int_{4\pi}I(s,\ \Omega')\cdot \Phi(\Omega'\ ;\ \Omega)\,d\Omega'. \quad (1b)$$

Here, $\beta(s)$ is the extinction coefficient of the medium and is the sum of the absorption coefficient $a(s)$ and the scattering coefficient $\sigma_s(s)$ as

$$\beta(s)=a(s)+\sigma_s(s) \quad (2)$$

The scattering phase function $\Phi(\Omega'\ ;\ \Omega)$ appearing in Eq. (1b) is the probability distribution function of the radiative energy scattered into the direction Ω that is incident from the direction Ω'. The scattering phase function is usually expressed in a series of the Legendre functions as [Clark et al. (1957)]

$$\Phi(\Omega'\ ;\ \Omega)=\sum_{n=0}^{K}C_nP_n(\cos\Psi) \quad (3)$$

Where the coefficients C_n in the series are determined from the Mie theory [Mie, 1908] which is derived by considering a spherical particle and are functions of the size parameter and the refractive index of the particle considered (Kim and Lee, 1988). In Eq. (3) Ψ indicates the angle measured between the incident and the scattered directions.

By integrating Eq. (1) along the path length s, we may obtain an apparent solution of the radiative transfer equation. The integration may be performed between an upstream point u, where the radiative intensity along the direction Ω is I_u, and a point P, where the intensity into Ω is I_P, as shown in Fig. 1. The resulting apparent solution is

$$I_P(s_P,\ \Omega)=I_u(s_u,\ \Omega)\exp\left\{-\int_{s_u}^{s_P}\beta(s)\,ds\right\} + \int_{s_u}^{s_P}S(s,\ \Omega)\ \exp\left\{-\int_{s}^{s_P}\beta(s')\,ds'\right\}ds \quad (4)$$

By assuming that the extinction coefficient $\beta(s)$ remains uniform within an infinitesimal distance $\Delta s=s_P-s_u$ as $\beta(s)\fallingdotseq\beta_P$, Eq. (4) can be written in a simpler form as

$$I_P(s_P,\ \Omega)=I_u(s_u,\ \Omega)\exp(-\beta_P\Delta s) + \int_{s_u}^{s_P}S(s,\ \Omega)\exp\{-(s_P-s)\beta_P\}ds \quad (5)$$

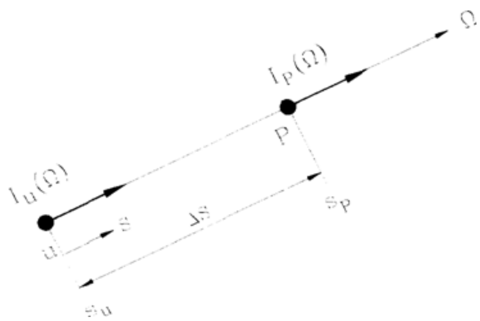


Fig. 1 Radiative intensity along a path.

As shown in Eq. (5) there still remains an integral over the path length in the apparent solution which needs further approximation for future numerical computations.

There are various ways of approximating the spatial integration which can result in fairly accurate intensity solutions. Some of these approsimations will be discussed in this section in some detail for comparison.

2.1.1 Taylor series approximation

To complete the spatial integration appearing in Eq. (5) the radiative source function $S(s, \Omega)$ can be expressed by the Taylor series expansion around a point P as (Chui and Raithby, 1993).

$$S(s, \Omega) = S_P(s_P, \Omega) + \left(\frac{\partial S(s, \Omega)}{\partial s}\right)_P (s - s_P) + \frac{1}{2!}\left(\frac{\partial^2 S(s, \Omega)}{\partial s^2}\right)_P (s - s_P)^2 + \cdots \quad (6)$$

In Eq. (6) we may neglect the high order terms of the infinitesimal distance $s - s_P$ and the following approximate expression for the radiative source function can be obtained.

$$S(s, \Omega) \fallingdotseq S_P(s_P, \Omega) + \left(\frac{\partial S(s, \Omega)}{\partial s}\right)_P (s - s_P) \quad (7)$$

Now Eq. (7) is substituted into the Eq. (5) and the intergration is performed analytically to obtain the following final form of the approximate apparent solution indicating the radiative transfer along the finite distance $\Delta s = s_P - s_u$.

$$I_P(s_P, \Omega) = I_u(s_u, \Omega)\, e^{-\beta_P \Delta s} + \frac{S_P(s_P, \Omega)}{\beta_P}(1 - e^{-\beta_P \Delta s}) - \frac{1}{\beta_P{}^2}\left(\frac{\partial S(s, \Omega)}{\partial s}\right)_P \{1 - e^{-\beta_P \Delta s}(1 + \beta_P \Delta s)\} \quad (8)$$

where the derivative of the radiative source function at point P can also be approximated by the slope of the source function between two neighbouring points P and u as

$$\left(\frac{\partial S(s, \Omega)}{\partial s}\right)_P \fallingdotseq \frac{S_P(s_P, \Omega) - S_u(s_u, \Omega)}{\Delta s} \quad (9)$$

2.1.2 Trapezoidal rule approximation

An another way of approximating the spatial integral is to approximate the integral directly by a numerical quadrature as the trapezoidal rule. That is, the integral can be replaced by the two-point quadrature expression between u and P, and the resulting approximate expression for the apparent solution can have the following form as

$$I_P(s_P, \Omega) = I_u(s_u, \Omega)\, e^{-\beta_P \Delta s} + \frac{\Delta s}{2}\{S_P(s_P, \Omega) + S_u(s_u, \Omega)\, e^{-\beta_P \Delta s}\} \quad (10)$$

Both of the Eqs. (8) and (10) can be used either for the FVM or the discrete ordinates interpolation method. The equation given in Eq. (10) has a much simpler form and is linearly dependent upon the medium transmittance for easier application to nongray gases as compared to Eq. (8).

2.2 Supplemental equations

As the boundary condition for the transfer equation shown in Eq. (8) or (10), we may consider the radiative intensity from a diffusely reflecting opaque wall as (Kim, 1995)

$$I_w(s_w, \Omega) = \varepsilon_w(s_w) I_{bw} + \frac{\rho_w(s_w)}{\pi}\int_{\vec{n}\cdot\Omega'<0} |\vec{n}\cdot\Omega'|\, I_w(s_w, \Omega')\, d\Omega' \quad \text{for } \vec{n}\cdot\Omega > 0 \quad (11)$$

where $\vec{n}$ is the unit inward normal vector from the wall.

Once the radiative intensity field is obtained from solving the radiative transfer equation, the average intensity (G) and the radiative heat fluxes can be obtained from the following expressions.

$$G(s) = \frac{1}{4\pi}\int_{\Omega=4\pi} I(s, \Omega)\, d\Omega \quad (12)$$

$$Q_{rx}(s)=\int_{\Omega=4\pi}\mu\, I(s,\ \Omega)\, d\Omega \tag{13a}$$

$$Q_{ry}(s)=\int_{\Omega=4\pi}\eta\, I(s,\ \Omega)\, d\Omega \tag{13b}$$

$$Q_{rz}(s)=\int_{\Omega=4\pi}\xi\, I(s,\ \Omega)\, d\Omega \tag{13c}$$

3. Discrete Ordinates Interpolation Method

3.1 Discrete ordinate equations

Solution of the radiative transfer can be obtained by using the approximate form of the radiative transfer equation shown in Eq. (8) or (10). For this purpose the equation of transfer should be solved at all locations and for all directions. The angular distribution of the radiative intensity is frequently approximated by considering the finite number of discrete ordinates which are the basis of the standard discrete ordinates method or the discrete ordinates interpolation method. Now we may apply the approximate form of the equation of transfer shown in Eq. (8) or (10) for these discrete ordinates which are established at a location in the medium for the total number of M discrete direction. For an arbitrary mth ordinate direction indicated by $\Omega_m=(\mu_m,\ \eta_m,\ \xi_m)$ we may write the discrete ordinate equation corresponding to the approximate equation of transfer as the following.

3.1.1 Taylor series expansion form

$$I_{P,m}=I_{u,m}e^{-\beta_P\Delta s_m}+\frac{S_{P,m}}{\beta_P}(1-e^{-\beta_P\Delta s_m}) -\frac{1}{\beta^2_P}\left(\frac{\partial S(s,\ \Omega)}{\partial s}\right)_{P,m}\{1-e^{-\beta_P\Delta s_m}(1+\beta_P\Delta s_m)\} \tag{14}$$

where,

$$\left(\frac{\partial S(s,\ \Omega)}{\partial s}\right)_{P,m}\doteqdot\frac{S_{P,m}-S_{u,m}}{\Delta s_m} \tag{15}$$

3.1.3 Trapezoidal rule form

$$I_{P,m}=I_{u,m}e^{-\beta_P\Delta s_m}+\frac{\Delta s_m}{2}\{S_{P,m}+S_{u,m}e^{-\beta_P\Delta s_m}\} \tag{16}$$

Here, the radiative source function for the mth discrete ordinate direction $S_{P,m}$ is also expressed in a discrete form by replacing the angular integral by the Gaussian quadrature as

$$S_{P,m}=a_PI_{bP}+\frac{\sigma_{sP}}{4\pi}\sum_{m=1}^{M}w_{m'}I_{m'}\Phi_{m';m} \tag{17}$$

where, $\omega_{m}{'}$ is the angular weight for the m'th ordinate direction.

In Eqs. (14)～(16) the radiative intensity ($I_{u,m}$) and the radiative source function ($S_{u,m}$) at the upstream u point can be determined by a suitable interpolation using the known upstream point values respectively.

3.2 Point u on the triangular interpolation plane

Figure 2 shows the typical node arrangement and the ordinate direction Ω in 3-D space where E, W, N, S, T and B are the neighbouring node points adjacent to the center node P. Similar node arrangement can be applied for any irregular grid system for the analysis suggested in this paper. The angular radiative intensity distribution a node P can be computed either by using Eq. (14) or Eq. (16). Prior to this calculation, the intensity I_u and source function S_u must be determined first by a suitable interpolation using the known intensity and source function values at the upstream nodes (W, S and B nodes for the concerned direction). In this study the linear interpolation scheme on a diagonally placed triangular plane (Seo and Kim, 1996) is used to determine I_u and S_u where the coordinates of the upstream interpolation point u can be determined by using the known edge coordinates of the triangle WSB.

The interpolation is performed on the triangle

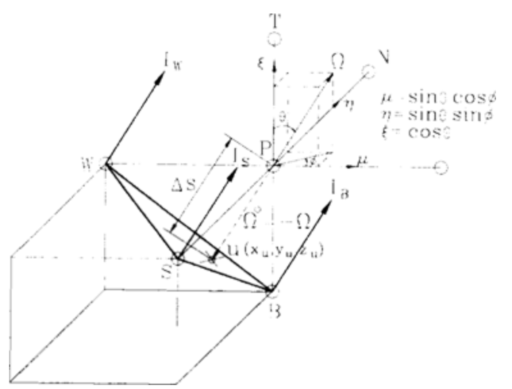


Fig. 2 Typical node arrangement for a triangular interpolation plane.

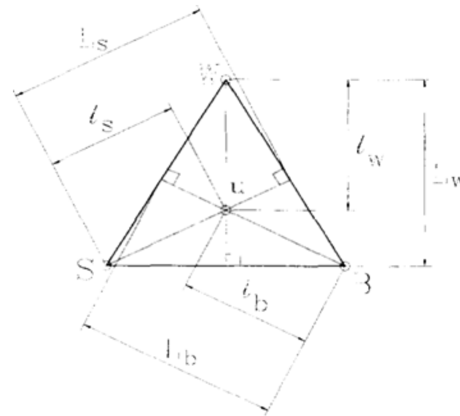


Fig. 3 Linear interpolation on a triangular element.

extending over the three upstream node points WSB where the known node values (ϕ_W, ϕ_S, ϕ_B) are specified. By referring to the same triangle shown flat on the page as in Fig. 3, an arbitrary function ϕ at the upstream point u (noted by ϕ_u) can be determined by considering the following linear interpolation as the inverse distance interpolation method (Ripley, 1981).

$$\phi_u=\left(1-\frac{l_w}{L_w}\right)\phi_W+\left(1-\frac{l_s}{L_s}\right)\phi_S+\left(1-\frac{l_b}{L_b}\right)\phi_B \tag{18}$$

where, $l's$ are the distances between the corner nodes and the u point, and $L's$ are the distances between the corner nodes and the edge lines.

4. Numerical Results and Discussions

To evaluate the accuracy of the approximate transfer equation, some typical radiative equilibrium problems are considered. For a radiative equilibrium we consider an energy balance equation.

$$\Delta\cdot\overrightarrow{Q_r}=\dot{q} \tag{19}$$

In this study the above radiative equilibrium is applied for a three dimensional regular parallelepiped and a three dimensional irregular enclosure.

4.1 Regular system

The System studied here has the dimensions of

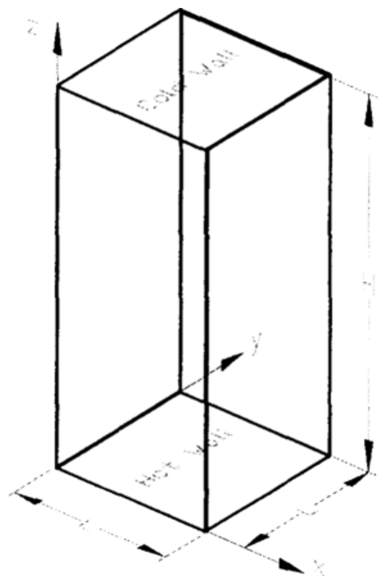
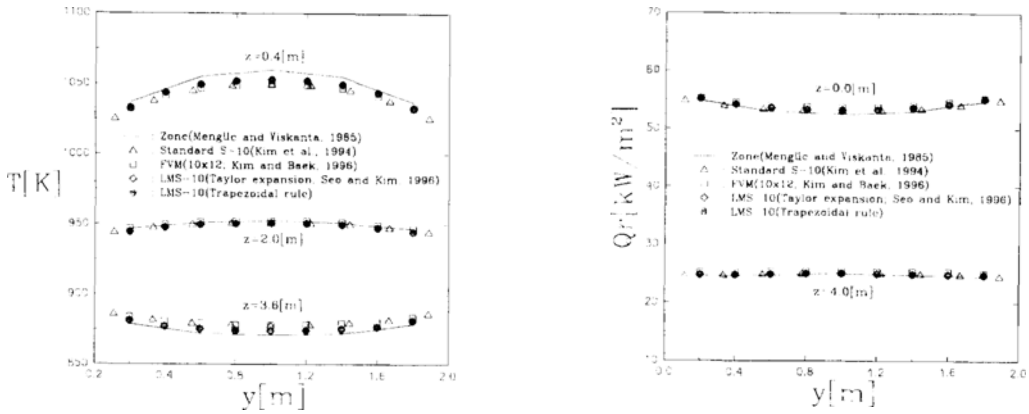


Fig. 4 Schematic diagram of the 3-D rectangular system.

$L\times L\times H$ ($L=2m$, $H=4m$) as shown in Fig. 4 (Mengüc and Viskanta, 1985). The bottom wall of the system ($z=0[m]$) is maintained at a constant temperature of $1200K$, with an emissivity of $\varepsilon_w=0.85$. The top wall ($z=4[m]$) is at $400K$ and $\varepsilon_w=0.7$. All the other side walls are at $900K$ and $\varepsilon_w=0.7$. The pure absorbing medium filling the system has the extinction coefficient of $\beta=0.5m^{-1}$ and has a uniform heat generation rate of $\dot{q}=5.0kW/m^3$. For the numerical study, the system is uniformly subdivided by $10\times10\times20$ control volumes.

Figure 5(a) and 5(b) show the comparison of the current LMS-10 results obtained from the discrete ordinates interpolation method using the trapezoidal rule with the results obtained from other methods. The LMS-10 (corresponding to the S-10 method) results using the Taylor series (Seo and Kim, 1996), the zonal method with $5\times5\times5$ control volumes (Mengüc and Viskanta, 1985), the standard S-10 method (Kim et al., 1994) and the FVM using the Taylor series with uniformly spaced 10×12 directions (Kim and Baek, 1997) are cosidered for the comparison. Figure 5(a) shows the temperature distributions obtained from the different methods, and the



(a) Temperature (b) Wall heat fluxes

Fig. 5 Comparison of temperatures and wall heat fluxes for various methods.

Table 1 Comparison of temperatures and wall heat fluxes at $x=y=1[m]$.

	$z[m]$	Zone	Errors relative to the zonal results			
			LMS-10 ($10\times10\times20$) Trapezoidal rule	LMS-10 ($10\times10\times20$) Taylor expansion	FVM(120) ($10\times10\times20$)	Standard S-10 ($10\times10\times20$)
T $[K]$	0.4$[m]$	1060.00	1053.20	1049.96	1049.8	
		error	−0.632%	−0.64%	−0.95%	−0.962%
	2.0$[m]$	953.00	951.25	951.15	952.79	951.16
		error	−0.184%	−0.194%	−0.02%	−0.193%
	3.6$[m]$	871.00	874.21	874.14	878.27	877.96
		error	0.369%	0.36%	0.83%	0.800%
Q_r $[kW/m^2]$	0.0$[m]$	52.50	53.22	53.26	53.70	53.00
		error	1.371%	1.45%	2.29%	0.952%
	4.0$[m]$	25.00	25.01	25.00	25.38	25.03
		error	0.04%	0.00%	1.52%	0.12%

(Note) $error=\left[\frac{\phi-\phi_{zone}}{\phi_{zone}}\right]\times100\%$

current LMS-10 results using the trapezoidal rule agree fairly well with those obtained from the other methods. The radiative heat flux distributions on the hot and cold walls are also compared in Fig. 5(b) with a good agreement with the other methods. In Table 1 the typical medium temperatures and the wall radiative heat fluxes shown in Fig. 5(a) and (b) are compared with the zonal results which are considered to be the benchmark solution. As listed in Table 1, the current LMS-10 results using the trapezoidal rule show a maximum relative error of 1.37% compared with zonal results. We may draw a conclusion that the proposed trapezoidal rule is satisfactory for the problem studied here.

4.2 Irregular system

To examine the applicability of the current trapezoidal rule on the more complicated three-dimensional systems, the radiative equilibrium in

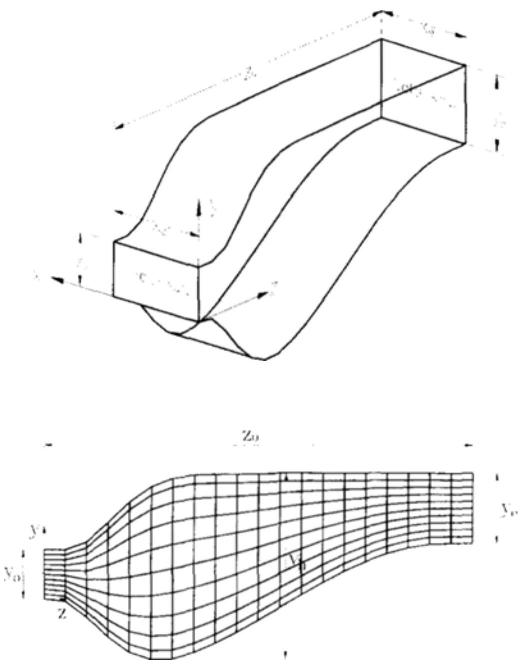


Fig. 6 Schematic diagram of the gas-turbine combustor considered.

Tabal 2 Typical dimensions of the gas-turbine combustor.

x directions	$x_0=x_e=2[m]$
y directions	$y_0=1\ [m]$, $y_e=1.423[m]$, $y_h=3.78\ [m]$
z directions	$z_0=7.6[m]$

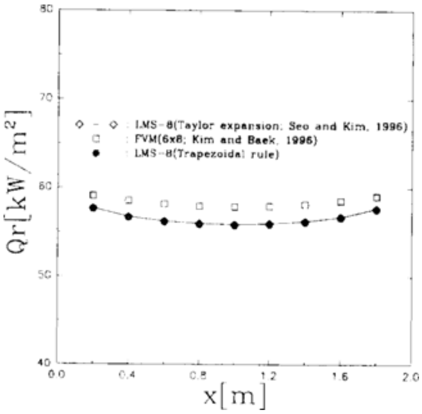
a complex gasturbine-combustor-shaped enclosure(Kim and Baek, 1996) as shown in Fig. 6 is considered. Wall temperatures, wall reflectivities and heat generation rate are assumed to be the same as those used in the previous regular system. Typical dimensions of the irregular system are given in Table 2. For numeircal computations, nonorthogonal grids are generated to result in $10\times10\times20$ nonuniform control volumes. The current results obtained from the discrete ordinates interpolation method with the trapezoidal rule are compared with those obtained from the FVM and the discrete ordinates interpolation method, both of which use the transfer equation approximated by the Taylor series expansion.

4.2.1 Effect of the scattering phase functions

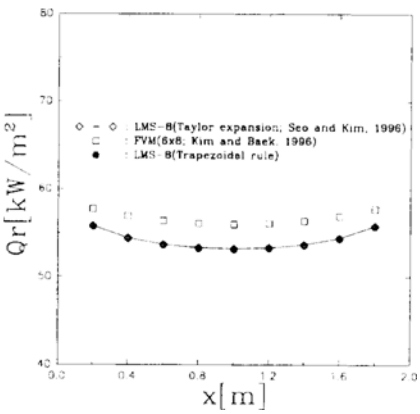
For the irregular system considered here the trapezoidal rule to the transfer equation is applied for an anisotropically scattering media to examine the effect of the scattering phase functions on the accuracy of this method. Three scattering phase functions studied here are the isotropic, forward scattering F2 and backward scattering B2 which are studied by Kim and Lee (1988). The extinction coefficient of the participating medium is $\beta=0.5m^{-1}$ and the scattering albedo of $\omega=0.7$. Figure 7 and 8 show the resulting radiative heat flux distributions on the hot and cold walls, respectively. For the isotropic, forward scattering F2 and backward scattering B2 phase functions, both of the LMS-8 results using the convensional Taylor series expansion and the current trapezoidal rule are matched fairly well on the wall heat flux results. However, the FVM results show a maximum error of 5.2% for B2 on the hot wall and 4.6% for B2 on the cold wall as compared to the LMS-8 results. Theoretically the LMS-8 results are considered to be more accurate than the FVM results obtained for the same numbers of spatial control volumes and discrete angular directions (Seo and Kim, 1996). The main reason of the inaccuracy experienced by the FVM is due to the use of the angular quadrature data generated for the uniformly divided discrete directions while the two LMS-8 methods use a more accurate Gaussian type integral quadrature.

4.2.2 Effect of the extinction coefficients.

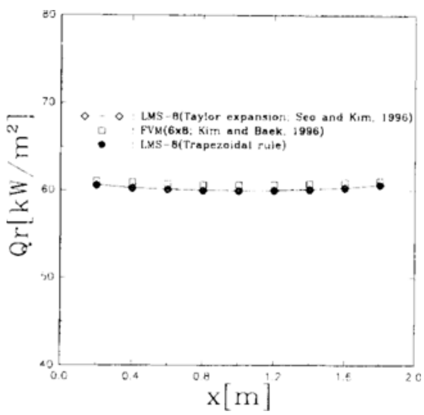
Another parameter considered for the comparison between the trapezoidal rule and the Taylor series is the medium extinction coefficient. For purely absorbing and emitting medium the radiative equilibrium is solved for three different extinction coefficients, $\beta=0.5$, 1.0, 2.0 $[m^{-1}]$. Figure 9 shows the comparison of the radiative wall heat fluxes obtained from the different methods for the different extinction coefficients considered. The results show that the wall heat fluxes are reduced as the medium extinction coefficient is increased and the trends are coincident for all the methods studied here. Among the



(a) Isotropic scattering

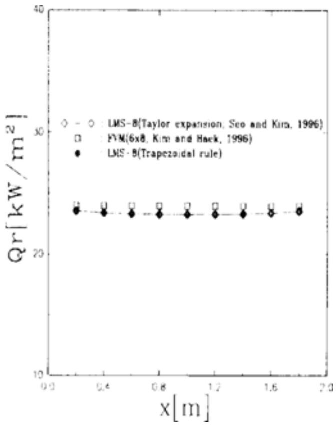


(b) Backward scattering(B2)

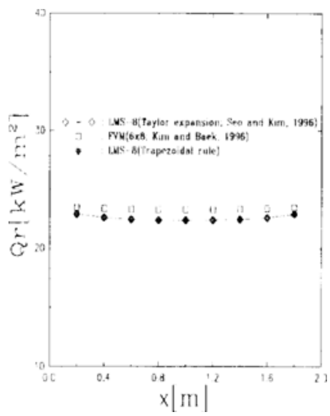


(c) Forward scattering(F2)

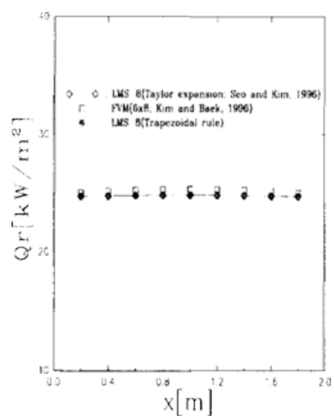
Fig. 7 Effect of the scattering phase functions on the hot wall heat fluxes.



(a) Isotropic scattering



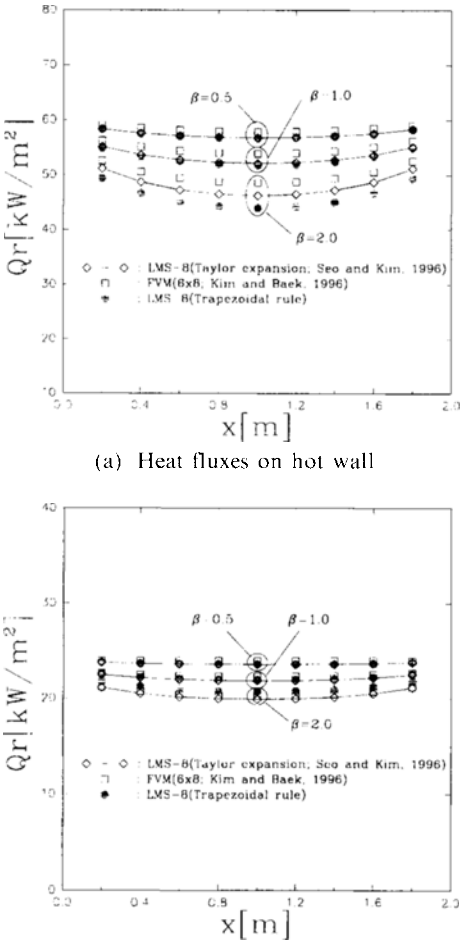
(b) Backward scattering(B2)



(c) Forward scattering(F2)

Fig. 8 Effect of the scattering phase functions on the cold wall heat fluxes.

methods compared, the results obtained from the LMS-8 method using both the trapezoidal rule and the Taylor series show a good agreement for the medium with a small extinction coefficient of



(a) Heat fluxes on hot wall

(b) Heat fluxes on cold wall

Fig. 9 Effect of the extinction coefficients on the hot wall and cold wall heat fluxes.

$\beta=0.5m^{-1}$. As the extinction coefficient is increased to $\beta=1.0$ or 2.0 $[m^{-1}]$ the discrepancy between two approximations is appreciable. However, even for the small extinction coefficient, the FVM results (Kim and Baek, 1996) deviate from those obtained from the two LMS-8 methods an appreciable amount, and for $\beta=2.0$ the absolute amounts of the error of the FVM from the LMS-8 method with the Taylor approximation are about the same order as that of the LMS-8 method with the trapezoidal rule. The errors observed here are about 2 to 3% for $\beta=1.0$ $[m^{-1}]$, and about 4 to 5% for $\beta=2[m^{-1}]$. For the participating media with very large extinction coefficients above $\beta=5.0m^{-1}$, the radiative equilibrium solution using the LMS-8 with the trapezoidal rule showed a very slow convergence for the rough grid system considered here. The Taylor series approximation did not show any difficulties for the same rough grids. The problem was overcome for the LMS-8 method by increasing the number of control volumes from $10\times10\times20$ to $20\times20\times30$.

5. Conclusion

In this paper an approximate solution of the radiative transfer equation using the two point trapezoidal rule which could be used in the solution methods developed for nonorthogonal grids such as the FVM and the discrete ordinates interpolation method is suggested and tested for some radiative equilibrium problems. The existing Taylor series approximation results in a complicated expression for the radiative intensity. The trapezoidal rule approximation to the apparent solution of the radiative transfer equation suggested in this study results in a simple expression which may be easily applied for more complicated radiative transfer problems with the relatively thin nongray gases.

The trapezoidal rule approximation to the apparent solution of the radiative transfer equation is applied to the regular and the irregular three dimensional enclosures by considering the radiative equilibrium with a constant volumetric heat generation. The trapezoidal rule is proved to be very satisfactory for the case of three dimensional radiative transfer with an anisotropic scattering media. However the method shows some difficulties in obtaining the convergence for the media with very large optical depths in which case finer grids are required.

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